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# Question Paper Code : 51407

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Fifth Semester

Computer Science and Engineering

MA 1251 — NUMERICAL METHODS

(Common to Information Technology and Electronics and Communication Engineering)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State fixed point theorem.

2. Compare the efficiency of Gauss-elimination and Gauss-Jordan methods for solving large size linear systems.

3. Compare the Lagrange's interpolation formula and Newton's forward difference formula.

4. Show that $\nabla^3 y_3 = \Delta^3 y_0$.

5. Show that $E = e^{hD}$.

6. Write down the three point Gaussian quadrature formula to approximate $\int_a^b f(x)dx$.

7. Compare Milne's method and Runge-Kutta fourth order method of solving an ordinary differential equation.

8. Write down a second order initial value problem and convert it into a first order coupled system.

9. Write down a finite difference approximation for the derivative $f'''(x)$.

10. Write down the two-dimensional Poisson equation governing a physical problem.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Use Regula-Falsi method to obtain a real root of the equation logx = cosx correct to four decimals. (8)

(ii) Solve the following system of equations by Gauss elimination method : (8)

$2x + y + z = 10;\ 3x + 2y + 3z = 18;\ x + 4y + 9z = 16$.

Or

(b) Solve the following system of equations by Gauss-Jacobi and Gauss – Seidel methods (five iterations) : $2x + 8y - z = 11$; $5x - y + z = 10; -x + y + 4z = 3$, with initial approximate solution $X^{(0)} = (0, 0, 0)^T$. (16)

12. (a) (i) Find a cubic Lagrange interpolating polynomial for the data: (0, – 3), (1,3), (2, 11) and (3, 27). (8)

(ii) A table of values of the function $f(x) = 2^x$ is given below. From the table find the value of $2^{1.1}$ using Newton's forward difference formula and also estimate an error for this computation : (8)

| | | | |
|---|---|---|---|
| $x$: | 1 | 2 | 3 |
| $f(x)$: | 2 | 4 | 8 |

Or

(b) (i) Using Newton's divided difference formula find $f(1.1)$ from the table: (8)

| | | | | | |
|---|---|---|---|---|---|
| $x$: | 1 | 2 | 3 | 4 | 5 |
| $f(x)$: | 2 | 4 | 8 | 16 | 32 |

(ii) Construct a natural cubic spline that passes through the points (-1, -1), (0,0) and (1, 1). (8)

13. (a) (i) Using Newton's difference method, find $f'(1)$ & $f'(4)$ from the table : (8)

| | | | | |
|---|---|---|---|---|
| $x$: | 1 | 2 | 3 | 4 |
| $f(x)$: | 2 | 4 | 8 | 16 |

(ii) Compute the integral $\int_0^{2.5} e^x\,dx$ by Trapezoidal rule and Simpson's 1/3rd rule with h = 0.5. Also compare with exact solution. (8)

Or

(b) (i) Using Romberg integration, evaluate $\int_0^{\pi} (\sin x)\,dx$ correct to four decimals.. (8)

(ii) Estimate $\int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$ as accurately as possible with $h = 1/4$. (8)

14. (a) Find $y(1.2)$, given $y''-xy'-y = \sin x$, $y(1) = 0$, $y'(1) = 0$ by Runge-Kutta method of order 4 with h= 1/5. (16)

Or

(b) Using Euler's method, solve the differential equation $y' = y = t^2 + 1$, $y(0) = \frac{1}{2}$ with $h = 0.2$ up to $y(0.6)$. Compare the results with exact solution. Also find $y(0.8)$ by Adam's predictor-corrector method.

(16)

15. (a) (i) Derive a finite difference scheme for solving a Poisson equation. (8)

(ii) Given the wave equation $u_{tt} = u_{xx}, 0 < x < 1, t > 0$ subject to the boundary conditions $u(0, t) = 0$ $u(1, t) = 0$ for $t > 0$ and the initial conditions $u(x, 0) = x - x^2, u_t(x, 0) = 0$ for $0 \le x \le 1$ by taking $h = k = 1/4$, compute the solution for the first 4 time steps. (8)

Or

(b) Use Crank-Nicolson scheme to find the solution of the following initial boundary value problem for one time step : $T_t = T_{xx}, 0 < x < 1, t > 0$ subject to the initial condition $T(x, 0) = x - x^2$ for $0 \le x \le 1$ and the boundary conditions $T(0, t) = T(1, t) = 0$ for $t > 0$. Compute the solution by taking $h = 1/4$ and $k = 0.025$. (16)

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